

Decoherence induced by an ordered environment

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To study the effect on the time evolution of a qubit, of an environment which undergoes a transition to an ordered phase, we consider a qubit weakly coupled to a standard BCS superconductor. We find that ordering in the superconducting bath generically leads to an unfavorable faster-than-exponential decay of the qubit coherence. On the other hand, a coupling to the non-ordered sector of the bath can result in a drastic reduction of the qubit decoherence. Since these behaviors are endemic to the ordered phase, qubits can serve as useful probes of phase transitions in their environment.

The past decade has seen tremendous activity devoted to developing experimentally viable qubits for quantum computing. These two-level systems are realized, for example, by directly using the charge or spin degrees of freedom of electrons and quantum dots [1] or more complex entities like flux qubits [2, 3] and Cooper boxes [4]. The utility of all these qubits for quantum computation is strongly limited by the influence of their environment which tends to destroy their quantum coherence. Consequently, a lot of recent theoretical studies have focused on ways and means of increasing the coherence time scales [5–10]. Another point of view on the interaction of small quantum systems with their surroundings consists in employing such ‘qubits’ as probes of the environment as typically done in Nuclear Magnetic Resonance spectroscopy [11]. This could be especially of use to investigate both equilibrium and non-equilibrium properties of cold atoms, for which standard thermodynamic measurements are difficult to carry out [12].

The first theoretical studies of decoherence considered environments consisting of independent harmonic oscillators [13] or spins [14, 15]. More recently, complex baths have been studied. In particular, intrabath interactions have been taken into account [6, 7, 16, 17], raising the question of the possible influence of thermodynamic phases and transitions on the decoherence of the qubit. Numerous results have been obtained but the complete picture is far from clear. It has been shown that the mitigating impact of intrabath interactions seen in many cases [16, 17] breaks down when the bath is in the vicinity of a continuous phase transition where the decoherence rate diverges on the disordered side [6]. Refs.16, 18 argued that symmetry breaking in the reservoir helps reduce decoherence, while Ref.17 found a strong Gaussian decay of quantum coherence. However, all these works suffer from different drawbacks ranging from a complete neglect of low energy modes in Ref.16 to obtaining an order parameter independent behavior for the time evolution of the qubit in Ref.17.

In this Letter, we revisit the problem of ordered baths to have a clearer understanding of their effect on qubits.

We consider a superconducting bath described by the Bardeen Cooper Schrieffer (BCS) theory. Though the BCS Hamiltonian does not capture the fluctuations in the disordered phase, it provides a very good description of the ordered phase. In line with Ref.6, where it was shown that the impact of the ordering on the qubit depends crucially on the relation between the qubit-bath interaction and the order parameter, we study different kinds of interactions between the qubit and the bath. The ordered superconducting phase is found to be characterized by a rich variety of behaviors not seen in the disordered phase, including faster-than-exponential decay of the coherence. The latter makes it unfavorable from the point of view of quantum computing. But, there are interesting exceptional qubit states which decohere slower when the bath orders. The sensitivity of the qubit to the order in the bath, makes it a good probe of the transition in the bath.

The combined system of the qubit and the superconducting bath is described by the Hamiltonian

$$H = \boldsymbol{\sigma}_q \cdot \mathbf{V} + H_B, \quad (1)$$

where $\boldsymbol{\sigma}_q$ is the vector Pauli operator for the qubit, whose components are the usual 2×2 Pauli matrices, \mathbf{V} is some bath vector operator that will be specified later, and H_B is the conventional BCS Hamiltonian $H_B = \sum_{k\epsilon} E_k \alpha_{k\epsilon}^\dagger \alpha_{k\epsilon}$ [19]. The Bogoliubov operators $\alpha_{k\epsilon}$ are related to the electron annihilation and creation operators by

$$\begin{aligned} \alpha_{k\epsilon}^\dagger &= u_k c_{k\epsilon}^\dagger + v_k c_{-k-\epsilon} \\ \alpha_{-k\epsilon} &= u_k c_{-k\epsilon} - v_k c_{k-\epsilon}^\dagger, \end{aligned} \quad (2)$$

where $c_{k\epsilon}^\dagger$ creates an electron with momentum k and spin $\epsilon = \uparrow, \downarrow$. The BCS dispersion relation is $E_k = \text{sgn}(e_k) \sqrt{e_k^2 + \Delta^2}$, where e_k is the underlying electronic dispersion and Δ is the superconducting gap. The coefficients in (2) obey $(u, v)_k^2 = (1 \pm e_k/E_k)/2$. We set $\hbar = k_B = 1$ in the rest of the paper. The superconducting order parameter Δ at temperature T is self-consistently

determined by

$$gN \int_0^{\omega_D} de \frac{\tanh(\sqrt{e^2 + \Delta^2}/2T)}{\sqrt{e^2 + \Delta^2}} = 1 \quad (3)$$

where g is the strength of the phonon-mediated electron-electron interaction, ω_D is the Debye frequency and N is the electronic density of states at the Fermi surface. As is well known, this equation determines a critical temperature T_c which separates a high-temperature metallic phase where $\Delta = 0$ and a low-temperature phase where Δ increases monotonically as T decreases. We assume that, at time $t = 0$, the qubit and the bath are uncorrelated and that the bath is in thermal equilibrium. The initial state of the combined system is thus $\Omega = \rho(0) \otimes \rho_B$ where $\rho_B \propto \exp(-H_B/T)$ and $\rho(0)$ is any qubit density matrix. The time evolution of the reduced density matrix of the qubit is given by $\rho(t) = \text{Tr}_B \Omega(t)$ where Tr_B denotes the partial trace over the bath degrees of freedom and $\Omega(t) = \exp(-itH) \Omega \exp(itH)$. In the limit of weak coupling between the qubit and the bath that we are interested in, one can use two main methods to calculate $\rho(t)$: i) the time-convolutionless (TCL) projection operator technique and ii) the Nakajima-Zwanzig (NZ) approximation [20, 21]. TCL gives local-in-time equations of motion for $\rho(t)$ whereas the NZ approximation gives an integro-differential dynamical equation for $\rho(t)$. The accuracy of these methods depends on the problem studied and it is difficult to assert a priori which one is more appropriate [22]. In this Letter, we focus on the asymptotic evolution predicted by the second-order TCL approximation, and briefly discuss the results obtained using the NZ technique at the end. To write the master equation given by the TCL approximation to second order, it is convenient to first rewrite the Hamiltonian (1) as $H = H_B + \boldsymbol{\sigma}_q \cdot \langle \mathbf{V} \rangle + H_I$ where $\langle \dots \rangle = \text{Tr}(\rho_B \dots)$ and $H_I = \boldsymbol{\sigma}_q \cdot (\mathbf{V} - \langle \mathbf{V} \rangle)$. With these notations, we obtain

$$\partial_t \rho = -i[\boldsymbol{\sigma}_q \cdot \langle \mathbf{V} \rangle, \rho(t)] - \int_0^t d\tau \text{Tr}_B [H_I, [H_I(-\tau), \rho(t) \otimes \rho_B]], \quad (4)$$

where $H_I(t) = \exp(itH_B) H_I \exp(-itH_B) = \boldsymbol{\sigma}_q \cdot (\mathbf{V}(t) - \langle \mathbf{V} \rangle)$. This equation is our starting point for the following calculations. We remark that, replacing the integral boundary t in (4) by $+\infty$ leads to the well-known Markovian master equation. However, as we will show below, the time evolution of $\rho(t)$ can be non-Markovian in the ordered phase precluding the use of such Markovian master equations.

Kondo coupling - We first consider a Kondo like coupling where the qubit couples to the electronic spin density at the origin $\mathbf{S}(0)$, i.e., $V_\alpha = \lambda S_\alpha(0) \equiv \lambda \sum_{k,k',\epsilon,\epsilon'} c_{k\epsilon}^\dagger \sigma_{\epsilon\epsilon'}^\alpha c_{k'\epsilon'}$ where $\alpha \in \{x, y, z\}$, λ is the coupling strength, and $\sigma_{\epsilon\epsilon'}^\alpha$ are the matrix elements of the Pauli matrix σ^α . The Hamiltonian (1) is effectively the same as that of a magnetic impurity embedded in a superconductor. Contrary to the situation of a metallic

bath where one really does not have a true weak coupling regime because of the dynamical Kondo effect, here we have a weak coupling regime [23]. The isotropy of the total Hamiltonian (1) and the absence of any net moment in the bath lead to the following simplifications: since $\langle \mathbf{V} \rangle = 0$, the first term in (4) vanishes and all components of the effective spin-1/2 corresponding to the qubit, i.e., $s_\alpha(t) = \text{Tr}(\rho(t) \sigma_q^\alpha)$, satisfy the same equation of motion. Consequently, the qubit evolution is characterized by a unique time function M defined by $s_\alpha(t) = M(t) s_\alpha(0)$ [7]. We see that with a Kondo like coupling, both decoherence and relaxation exhibit the same time evolution. By writing the electron operators $c_{k\epsilon}$ in terms of the Bogoliubov quasi-particle operators (2), we find

$$\ln M(t) \simeq -8\lambda^2 \int d\omega \frac{\sin(\omega t/2)^2}{\omega^2} S^+(\omega), \quad (5)$$

with the dynamical spin structure factor S^+ given by

$$S^\pm(\omega) = \int de f^\pm(e, \omega) \rho(e) \rho(e - \omega) n(e) n(\omega - e), \quad (6)$$

where $f^\pm(e, \omega) \equiv 1 \pm \Delta^2/e(e - \omega)$, $\rho(e) = |e|(e^2 - \Delta^2)^{-1/2}$ is the superconducting density of states, and $n(e)$ is the Fermi function.

For $T > T_c$, the bath is a simple metal ($f^\pm(e) = 1$) and we obtain the usual asymptotic Markovian decay $\ln M(t) = -\gamma t$ with a decay rate $\gamma = 4\pi\lambda^2 S^+(0)$ which increases and saturates to a density of states dependent value at very high temperatures [7, 8]. As one approaches the transition temperature $T \rightarrow T_c^+$, we expect the growing fluctuations to result in a divergent rate γ at T_c [6] though this is not captured by the mean field BCS theory used here. We now analyze the asymptotic qubit evolution in the ordered phase $0 \leq T < T_c$. At $T = 0$, we find that $S^+(\omega) = 0$ for all $\omega > -2\Delta$, where 2Δ is the gap to two particle excitations. This gap leads to an incomplete (power-law) decoherence of the qubit just as seen in the case of an insulating bath [8]. At temperature $0 < T < T_c$, due to the divergence present in the superconducting density of states, S^+ is infra-red divergent: $S^+(\omega) \simeq -r(T) \ln|\omega/T|$ as $\omega \rightarrow 0$. The pre-factor $r(T) = \Delta/2 \cosh^2(\Delta/2T)$, shown in the inset of Fig.1, is a non-monotonic function of temperature which vanishes at $T = 0$ and $T = T_c$. This infrared divergence results in

$$\ln M(t) \simeq -2\pi\lambda^2 r(T) t \ln t \quad (7)$$

for times $t \gg t_a \equiv \text{Max}(1/T, 1/\Delta)$. We see that, contrary to naive expectations, the ordered bath leads to a novel faster-than-exponential loss of coherence of the qubit. This can be attributed to the fact that the qubit couples to the spin fluctuations and hence order parameter fluctuations via the singlet Cooper pairs. Moreover, we see an interesting reentrance in the asymptotic regime because the coefficient $r(T)$ which dictates the asymptotic decoherence is the same for two different temperatures (cf Fig. 1).

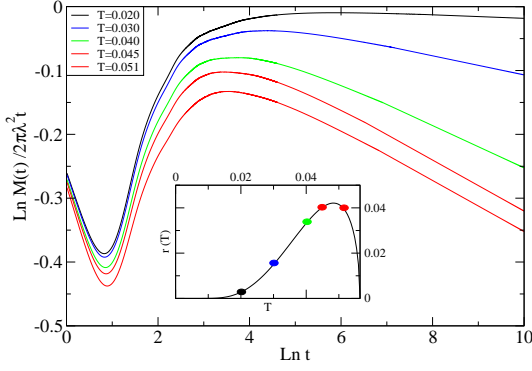


FIG. 1: $\ln M(t)/t$ as a function of $\ln t$ for $T = 0.02, 0.03, 0.04, 0.045$ and 0.051 in units of ω_D . The inset shows the coefficient r as a function of T . Here $gN = 0.33$, $T_c \simeq 0.056\omega_D$ and $\Delta(0) \simeq 0.1\omega_D$.

Order coupling - We now consider the coupling described by $V_z = 0$, $V_x = V + V^\dagger$ and $V_y = iV - iV^\dagger$ where $V = \lambda \sum_{k,k'} c_{k\uparrow}^\dagger c_{k'\downarrow}$ is the order parameter at the origin. Such couplings are relevant for qubits made from Cooper pair boxes [4, 24]. In this case, we obtain, from (4), the following set of coupled differential equations for the components s_α :

$$\partial_t \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = \begin{pmatrix} \Sigma_-(t) & 0 & 0 \\ 0 & \Sigma_+(t) & h \\ 0 & -h & \Sigma_-(t) + \Sigma_+(t) \end{pmatrix} \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} \quad (8)$$

where $\Sigma_\pm(t) = -8\lambda^2 \int d\omega \sin(\omega t) S^\pm(\omega)/\omega$ and $h \equiv -2\langle V_x \rangle = 2\lambda\Delta/gN$. Unlike the Kondo case previously studied, here the equations for the components y and z are coupled in the ordered phase where $\Delta \neq 0$. A closer look shows that Σ_- and hence s_x are related to the dynamic charge correlation function which is non-singular and Σ_+ to the spin correlation function which shows singular behavior. As will be discussed below, this leads to a complex time evolution of the qubit.

For $T > T_c$, all the components are uncoupled and decay asymptotically as $\ln s_\alpha = -\gamma_\alpha t$ with the rates $\gamma_x = \gamma_y = 8\pi\lambda^2 S^-(0)$ and $\gamma_z = 2\gamma_x$. The existence of two different rates is a direct consequence of the spin anisotropy of the order coupling whereas in the spin isotropic Kondo case, all rates are the same. In the ordered phase, at $T = 0$, solving Eq.8, we find that the presence of a gap, both in Σ_- and Σ_+ , leads to an incomplete decay of the central spin. For $T \neq 0$, S^- is regular at low frequencies which results in a Markovian decay $\ln s_x \simeq -\gamma_x t$ for times $t \gg t_a$. The full temperature dependence of the rate γ_x is shown in the inset of Fig. 2. Its behavior for $T \rightarrow 0$ is $\gamma_x \propto T e^{-\Delta/T}$. This is an important result of our work because it shows that in the ordered phase, the Markovian rate for the component s_x is strongly suppressed compared to a simple metallic bath. This is very similar to the relaxation induced by a coupling to the charge fluctuations, studied in the context of NMR by Fulde and Black [25], which

is described by $\sigma_q \cdot \mathbf{V} \propto \sigma_z \hat{n}$ where \hat{n} is the number of electrons at the origin. For $T \geq T_c$, the rate γ_x coincides with the rate of the equivalent metallic bath with $\Delta = 0$ and saturates to a finite DOS-dependent value proportional to $\int dE \rho(E)^2$ as expected. If the qubit's entire evolution was determined by this component, then the bath is effectively a semiconductor with a temperature dependent gap [8]. On the other hand, we find that the components s_y and s_z exhibit non-Markovian behavior. In the limit of weak coupling between the qubit and the bath, Eq.(8) can be solved for s_y and s_z [26]. By writing $(s_y, s_z) = (s_y(0), s_z(0)) \exp(A)$ and keeping only order terms of the 2×2 matrix A , we find

$$\begin{aligned} s_y &\simeq e^{a_1} \left[s_y(0) (a_0 \text{sinc } \Theta + \cos \Theta) + s_z(0) h t \text{sinc } \Theta \right] \\ s_z &\simeq e^{a_1} \left[-s_y(0) h t \text{sinc } \Theta + s_z(0) (\cos \Theta - a_0 \text{sinc } \Theta) \right] \end{aligned} \quad (9)$$

where $a_\mu(t) = \int_0^t dt' [\mu \Sigma_+(t') + (\mu - 1/2) \Sigma_-(t')]$, $\Theta(t) = [(ht)^2 - a_0(t)^2]^{1/2} \simeq ht$, and $\text{sinc } \Theta = \Theta^{-1} \sin \Theta$. Note that both s_y and s_z show oscillatory behavior in the ordered phase with a frequency proportional to the order parameter. At $T = 0$, s_y and s_z oscillate at a maximal frequency and at finite temperatures $0 < T < T_c$, these oscillations are damped with the envelope $a_\mu(t) \propto -r(T)t \ln t$ for $t \gg t_a$ and we recover the faster-than-exponential decay (7) with reentrant temperatures encountered for the Kondo coupling (cf Fig. 2). The appearance of oscillations of the qubit can be used as a tool to demarcate the phase diagram, measure the gap and the critical temperature of the superconductor.

Because of the very different asymptotic behaviors of the components s_α found above, the decay timescale of the qubit depends crucially on the initial conditions. Assume the qubit is initially prepared in an eigenstate of σ_x . Then the components $s_y(t) = s_z(t) = 0$ are constant, and the asymptotic time evolution of the qubit, determined by $s_x(t)$, is non-oscillatory and Markovian with a highly reduced rate in the ordered phase. These pure states are thus relatively stable in the environment considered here. Moreover, a combination of good initial preparation and pulse sequences which repeatedly orient the qubit in the x direction could efficiently reduce decoherence times [27, 28]. For an initial qubit state such that $s_x(0) = 0$, this component remains zero and the asymptotic behavior of the qubit consists of an oscillatory and faster-than-exponential decay. In this case, due to the coupling between y and z components, we have situations where even if one of the components s_y or s_z is initially zero, this component can grow with time and show oscillatory behavior. For generic initial states, since s_y and s_z decay much faster than s_x , the asymptotic time evolution of the qubit is Markovian. The qubit state first decoheres into a statistical mixture of the eigenstates of σ_x and then relaxes exponentially into the maximally mixed state.

In both the Kondo and order coupling cases, we find

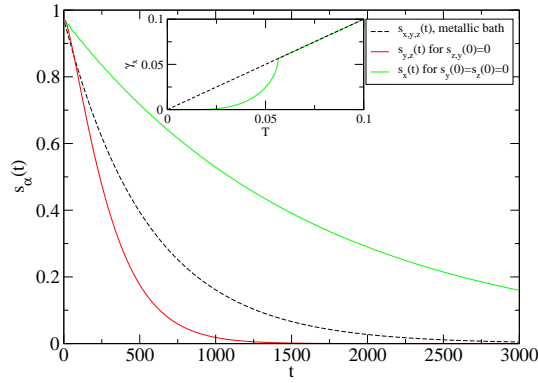


FIG. 2: $s_\alpha(t)$ as a function of t for the order coupling to a superconducting bath at $T = 0.045\omega_D$ for the initial conditions indicated in the legend. For the components s_y and s_z , only the envelope is plotted. The inset shows γ_x as a function of T for metallic and superconducting baths. Here $gN = 0.33$ and $\lambda = 0.05$.

interesting intermediate time behaviors but these will be discussed elsewhere [29]. We now briefly discuss the results obtained using the NZ approximation [8]. The conclusions for the Markovian behavior seen for the component s_x in the order coupling case remain unchanged. However, the asymptotic faster-than-exponential behavior seen in both the Kondo and order coupling cases is replaced by a much slower non-Markovian behavior $M_{NZ}(t) \sim \int d\omega \cos(\omega t) / \ln |\omega| \sim 1/t \ln t$ for times $t \gg t_{nz}$. We find $t_{nz} \gg t_a$ and an intermediate regime $t_a \ll t \ll t_{nz}$ characterized by a quantitatively faster decay than that predicted by the TCL approach, where the qubit becomes practically incoherent. Anomalously fast decoherence seems to be a feature of both NZ and TCL methods in the ordered phase.

To summarize, we have studied the influence of a true long-range ordered bath on the state of a qubit. We found, for two different qubit-bath couplings, a faster-than-exponential decoherence of the qubit in the ordered phase leading to the conclusion that ordered baths are disastrous for qubits. However, for one of the couplings considered, some particular pure states of the qubit are better preserved from decoherence when the bath orders, making such states potentially useful in experiments using echo sequences or other related techniques. This result constitutes another example of the important role played by the form of the coupling to a bath that can order [6]. We expect, if fluctuations in the disordered phase are properly taken into account, a divergence of the Markovian rate at the transition from the disordered side, followed by a faster-than-exponential loss of coherence in the ordered phase for generic initial states of the qubit. Exceptions are possible, as our study shows. We believe that this picture should be valid for all ordered baths provided the qubit couples in some way to the order parameter. It would be interesting to generalize our model to a non-degenerate qubit and to study the effect of such anomalous dissipation on the tunneling of the

qubit [13].

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